Reg. No. | | | | | | | | |

G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.

UG DEGREE END SEMESTER EXAMINATIONS - NOVEMBER 2024.
(For those admitted in June 2023 and later)

PROGRAMME AND BRANCH: B.Sc., STATISTICS

SEM	CATEGORY	COMPONENT	COURSE CODE	COURSE TITLE
III	PART – III	CORE-5	U23ST305	ESTIMATION THEORY

Date & Session:09.11.2024 / AN **Time : 3 hours** **Maximum: 75 Marks**

Course Outcome	Bloom's K-level	Q. No.	SECTION – A (10 X 1 = 10 Marks) Answer ALL Questions.
CO1	K1	1.	Which of the following is true about an unbiased estimator? a) Its variance is always zero. b) Its expected value equals the true value of the parameter being estimated. c) It always provides a more efficient estimate than biased estimators. d) It can only be used with maximum likelihood estimation.
CO1	K2	2.	According to the Neymann Factorization Theorem, a statistic is sufficient for a parameter if: a) It minimizes the mean squared error. b) The likelihood function can be factored into a product of two functions, where one depends only on the data and the parameter. c) It is consistent and unbiased. d) It is based on a Gaussian distribution.
CO2	K1	3.	Which of the following statements is true about a Minimum Variance Unbiased Estimator (MVUE)? a) MVUE always has the lowest possible variance among all estimators. b) MVUE must satisfy both unbiasedness and efficiency. c) MVUE is always unique for all distributions. d) MVUE minimizes variance among all unbiased estimators.
CO2	K2	4.	The Rao-Blackwell Theorem helps in: a) Finding the Maximum Likelihood Estimator (MLE). b) Reducing the variance of an unbiased estimator by conditioning on a sufficient statistic. c) Proving that an estimator is biased. d) Comparing the efficiency of two different biased estimators.
CO3	K1	5.	Which of the following is a key feature of the Maximum Likelihood Estimation (MLE) method? a) It finds parameters by equating sample moments to population moments. b) It is always an unbiased estimator. c) It maximizes the probability (or likelihood) of observing the given sample data. d) It guarantees the lowest possible variance for small sample sizes.
CO3	K2	6.	In the Method of Moments, parameter estimates are obtained by: a) Maximizing the log-likelihood function. b) Minimizing the sum of squared residuals. c) Using Bayesian techniques to compute posterior probabilities. d) Solving equations where sample moments are set equal to population moments.

CO4	K1	7.	Which of the following is an assumption made in the Method of Least Squares for linear regression? a) The errors are dependent and follow a uniform distribution. b) The relationship between the dependent and independent variables is non-linear. c) The errors have constant variance (homoscedasticity) and are normally distributed. d) The estimator is always unbiased regardless of the sample size.
CO4	K2	8.	The Method of Minimum Chi-Square is often used in: a) Estimating parameters for continuous probability distributions. b) Minimizing the sum of squared errors in regression analysis. c) Fitting models to data when working with categorical or frequency data. d) Maximizing the likelihood function for parameter estimation.
CO5	K1	9.	Which of the following is true about the confidence interval for the mean of a normal population when the population variance is known? a) The t-distribution is always used to construct the confidence interval. b) The Z-distribution is used, and the interval is centered around the sample median. c) The Z-distribution is used, and the interval is centered around the sample mean. d) Confidence intervals can only be constructed for small samples.
CO5	K2	10.	When constructing a confidence interval for the difference between the means of two normal populations with large samples, which of the following assumptions is necessary? a) The sampling distributions of the sample means are approximately normal. b) The two populations must have the same variance. c) The sample sizes must be equal. d) The samples must be drawn from populations with identical means.
Course Outcome	**Bloom's K-level**	**Q. No.**	**SECTION – B (5 X 5 = 25 Marks)** **Answer ALL Questions choosing either (a) or (b)**
CO1	K3	11a.	State and proof the invariance property of consistent estimators. **(OR)**
CO1	K3	11b.	Let $x_1, x_2, \ldots, x_n$ be a random sample from $N(\mu, \sigma^2)$ population. Find the sufficient estimators for μ & σ^2.
CO2	K3	12a.	Show that, an MVUE is unique. **(OR)**
CO2	K3	12b.	Let T1 and T2 be unbiased estimators of $\gamma(\theta)$ with efficiencies e1 and e2 respectively and $\rho = \rho_\theta$ be the correlation coefficient between them. Then $\sqrt{e_1 e_2} - \sqrt{(1-e_1)(1-e_2)} \le \rho \le \sqrt{e_1 e_2} + \sqrt{(1-e_1)(1-e_2)}$
CO3	K4	13a.	State the properties of MLE. **(OR)**
CO3	K4	13b.	Find the moment estimator of Exponential distribution with parameter θ.
CO4	K4	14a.	State the key assumptions made in the Method of Least Squares for estimation in a simple linear regression model. **(OR)**
CO4	K4	14b.	Briefly explain the Method of Minimum Chi-Square and mention one area where this method is applied.
CO5	K5	15a.	Explain the concept of a confidence interval. How is the confidence interval for the mean of a normal population calculated when the population variance is known? **(OR)**
CO5	K5	15b.	How do you calculate a confidence interval for a population proportion for large samples? Briefly explain with the formula.

Course Outcome	Bloom's K-level	Q. No.	**SECTION – C (5 X 8 = 40 Marks)** **Answer ALL Questions choosing either (a) or (b)**
CO1	K3	16a.	State and prove the sufficient conditions for consistency.
			(OR)
CO1	K3	16b.	A random sample X_1, X_2, X_3, X_4, X_5 of size 5 is drawn from a normal population with unknown mean μ; (i) $t_1 = \frac{X_1+X_2+X_3+X_4+X_5}{5}$ (ii) $t_2 = \frac{X_1+X_2}{2} + X_3$ (iii) $t_3 = \frac{2X_1+X_2+\lambda X_3}{3}$ where λ is such that t_3 is an unbiased estimator of μ. Find λ; Are t_1 and t_2 unbiased? State giving reasons, the estimator which is best among t_1, t_2 and t_3.
CO2	K4	17a.	State and prove the Cramer-Rao inequality.
			(OR)
CO2	K4	17b.	State and prove the Rao Blackwell Theorem.
CO3	K4	18a.	In a random sample from normal population $N(\mu,\sigma^2)$ to find the maximum likelihood estimator for the first case (i) μ when σ^2 is known (ii) σ^2 when μ is known.
			(OR)
CO3	K4	18b.	A random variable X takes the values 0,1,2 with respective probabilities $\frac{6}{4N}+\frac{1}{2}\left(1-\frac{\theta}{N}\right), \frac{\theta}{2N}+\frac{\alpha}{2}\left(1-\frac{\theta}{N}\right), \frac{\theta}{4N}+\frac{1-\alpha}{2}\left(1-\frac{\theta}{N}\right)$ where N is a known number and α *and* θ are unknown parameters. If 75 independent observations on X give the values 0,1,2 with frequencies 27,38,10 respectively. To estimate, α *and* θ by using method of moments.
CO4	K5	19a.	Derive the least squares estimators for the slope and intercept in a simple linear regression model. Clearly state the assumptions required for these estimators to be optimal.
			(OR)
CO4	K5	19b.	Explain the Method of Minimum Chi-Square for parameter estimation and compare it with the Method of Least Squares. Provide an example where the Minimum Chi-Square method is more appropriate than the Least Squares method.
CO5	K5	20a.	Derive the formula for the confidence interval for the difference between the means of two normal populations when the population variances are known. Explain the assumptions made during this process.
			(OR)
CO5	K5	20b.	Explain how to construct a confidence interval for the mean of a normal population when the population variance is unknown. Discuss how the interval changes for large and small samples, and mention the distributions involved.